**Validating the Adaptive Scalaron Model (RFT 9.0)**

**Targeted Simulation Expansion**

**Dwarf Galaxy Formation and Core Evolution**

We perform ultra-high-resolution cosmological simulations of structure formation with the **adaptive scalaron** (ultralight axion) dark matter to capture dwarf galaxy assembly in detail. Using a dedicated Schrödinger–Poisson solver (e.g. the *SCALAR* code)​file-pbs5tcrmsvz7ndprsed51h, we evolve a representative volume from early times to $z\sim0$. **Resolution is key:** we achieve sub-kiloparsec (tens of parsecs) resolution to resolve the **solitonic cores** that form at the centers of dwarf-mass halos due to quantum pressure support​file-pbs5tcrmsvz7ndprsed51h. Prior simulations (for $m\_a \sim 2×10^{-22}$ eV) showed the emergence of long-lived, high-density soliton cores in halos​file-pbs5tcrmsvz7ndprsed51h. These cores give dwarf galaxies **shallower central density profiles** than the sharp cusps of $\Lambda$CDM and can even produce a mild excess bump in the inner rotation curve (a distinct kinematic signature)​file-pbs5tcrmsvz7ndprsed51h. In our new runs, we measure each dwarf’s core density, radius, and halo mass to test the **core–halo scaling relation** predicted by fuzzy dark matter theory. We expect the core size to scale inversely with halo mass (approximately $r\_c \propto M\_{\rm halo}^{-1/3}$)​file-pbs5tcrmsvz7ndprsed51h, and we will verify if this relation holds across formation histories. We also track cores through mergers: when two halos merge, their solitons interfere and eventually a new, heavier core settles in the remnant halo​file-9mbj5mvtgwzrvhmxttwfgp. We check whether the **soliton scaling law persists after mergers** (as previous studies suggest)​file-pbs5tcrmsvz7ndprsed51h​file-pbs5tcrmsvz7ndprsed51h. Any deviation (e.g. an oversized core or core oscillations post-merger) would refine the model’s predictions for dwarf galaxy density profiles.

To visualize core formation, the simulations output **density maps** and rotation curves over time. We monitor how a initially low-density protogalaxy develops a solitonic core as it accretes matter. The scalaron field’s quantum pressure counteracts gravity in the inner kiloparsec, preventing runaway cusp collapse. By $z\sim3$, nascent dwarf halos in the simulation exhibit well-formed cores of size $\sim1$ kpc (for $m\_a \sim 10^{-22}$ eV), consistent with theoretical expectations​file-pbs5tcrmsvz7ndprsed51h. We record the **time evolution of core properties**, finding that soliton cores grow in mass until reaching a quasi-equilibrium. Modest **core oscillations** are expected as the core exchanges energy with the granular outer halo; these will be quantified by tracking density fluctuations at the center. Detecting any coherent oscillation mode would be an interesting signature of the scalar field (a “breathing” core). Overall, the dwarf-scale simulations will provide concrete predictions for **core sizes, densities, and their redshift evolution**, which we will later compare to observed dwarf galaxy cores.

**Satellite Halo Disruption in Tidal Fields**

In parallel, we simulate a Milky Way–mass halo to study the fate of **satellite subhalos** under both cold DM and scalaron DM, focusing on tidal disruption and implications for stellar streams. In the scalaron model, low-mass subhalos are less abundant and each is **less concentrated** (since it contains a central soliton core rather than a steep cusp)​file-pbs5tcrmsvz7ndprsed51h​file-pbs5tcrmsvz7ndprsed51h. We populate the host halo with dwarf-mass subhalos and orbit them through its tidal field (including a static disk potential for realism). The goal is to see how quickly fuzzy-dark-matter subhalos **lose mass and disrupt** compared to $\Lambda$CDM subhalos. A cored subhalo is more vulnerable to tides – as it approaches the host, its outer layers strip off and even the core can be pried apart if tidal forces exceed the soliton’s self-gravity. This addresses the *too-big-to-fail* problem: in CDM, the most massive subhalos are too dense (cusp cores) to be consistent with the observed dwarfs, but in the scalaron model those same subhalos develop large cores and can be tidally pruned down to the right masses. We will quantify this by measuring **subhalo density profiles and mass loss rates** over time.

Crucially, we vary the scalaron particle mass to see its impact on substructure survival. Lower $m\_a$ (more “fuzzy” dark matter) suppresses small halos more strongly​file-pbs5tcrmsvz7ndprsed51h, potentially solving small-scale issues but risking an underproduction of satellites. For example, an extremely low mass $m\_a < 4×10^{-23}$ eV would likely **over-suppress satellite galaxies** (too few surviving subhalos)​file-pbs5tcrmsvz7ndprsed51h, which is in tension with the number of dwarf satellites observed. Our simulations bracket this regime by running cases with $m\_a$ from $10^{-23}$ up to $10^{-21}$ eV​file-pbs5tcrmsvz7ndprsed51h, showing the continuum from strong suppression (few satellites, all with big cores) to weak suppression (many satellites, smaller cores). We count the **subhalo mass function** in each run and track, for instance, how many subhalos above $10^8,M\_\odot$ remain at $z=0$. This will be directly comparable to Milky Way satellite counts and to substructure lensing observations. We also examine the **internal structure** of surviving subhalos: a subhalo in the fuzzy run might retain a soliton of radius hundreds of parsecs, which could be detectable as a low-density core in dwarf satellites of the Milky Way.

To connect with **stellar streams** (like Palomar 5 and GD-1), we integrate mock stellar streams in the simulated Milky Way halo. We initialize a thin stream of tracer particles and let it evolve in the live scalaron halo potential. The time-varying granular density of the fuzzy halo (mass clumps of order $\sim10^6$–$10^7,M\_\odot$ for $m\sim10^{-22}$ eV) will continuously perturb the stream​file-pbs5tcrmsvz7ndprsed51h. We measure effects such as **stream thickening, gaps or spur formation, and velocity kicks** imparted to stream stars. Even if an entire subhalo is disrupted, its remnant soliton core (or just the ambient fluctuations of the host halo) can leave subtle imprints on streams. By comparing a control run (smooth potential) to the scalaron run, we identify distinctive patterns: Fuzzy DM’s perturbations are expected to be **gentle and frequent** (many overlapping wave disturbances), in contrast to CDM where streams are mostly smooth until a rare **hard hit by a dense subhalo**​file-pbs5tcrmsvz7ndprsed51h​file-pbs5tcrmsvz7ndprsed51h. Our analysis will produce “heat maps” of stream density and thickness over length, as well as a **power spectrum of stream perturbations**​file-pbs5tcrmsvz7ndprsed51h. These simulated perturbation profiles will later be matched against Gaia data for real streams. If the scalaron model is correct, we anticipate a continuous wobble in the stream rather than isolated large gaps – a prediction we will test observationally. The **minimum axion mass** that keeps streams very thin can be inferred: e.g. if a $1.5×10^{-22}$ eV run shows noticeable stream distortion, then the real Pal 5 thinness (if undisturbed) would imply $m\_a > 1.5×10^{-22}$ eV​file-pbs5tcrmsvz7ndprsed51h.

**Scalaron Collapse Thresholds and Wave Emission**

An intriguing aspect of scalar field dark matter is the possibility of **gravitational collapse** of overdense regions, leading to transient phenomena. In the simulations, we investigate what happens if a solitonic core grows beyond its stable mass limit. Pure fuzzy dark matter (with only gravity and quantum pressure) has a known maximum mass for a stable soliton (analogous to the Chandrasekhar limit for a boson star). If a halo’s central soliton exceeds this threshold – for $m\_a \sim 10^{-22}$ eV, the threshold is on the order of $M\_{\rm crit} \sim 10^8$–$10^9,M\_\odot$ (comparable to the largest expected soliton in a cluster-sized halo) – the core can become **dynamically unstable**. We will simulate idealized collapses by gradually loading a soliton with extra mass (for example, by merging multiple cores rapidly or imposing a perturbation) and follow the outcome. Do we get a runaway collapse to a black hole, or does the soliton “explode” and shed mass? Relativistic simulations in the literature indicate that an axion star exceeding $M\_{\rm crit}$ does *not* immediately form a black hole; instead, it experiences a violent collapse until higher-order repulsive effects or quantum pressure halt it, after which a large fraction of its mass is expelled as **scalar waves** (relativistic axions)​[arxiv.org](https://arxiv.org/pdf/1608.06911#:~:text=states%2C%20as%20compared%20to%20the,rapid%20emission%20of%20relativistic%20axions). In our cosmological context (which is non-relativistic), we won’t form an actual black hole, but we can capture the analogue of that phenomenon: a near-critical soliton should eject mass in the form of **dispersive wave pulses** propagating outward.

We will analyze the **collapse threshold** by identifying the point at which a soliton’s oscillations grow without bound. For instance, as two massive halos merge, their solitons coalesce; the merged soliton might temporarily overshoot the stable mass. We expect to see rapid oscillations of the core and large amplitude **density fluctuations**. The code will record the energy carried away by scalar field waves radiating out of the core region. We will characterize the **wave emission profile**: its frequency spectrum, duration, and anisotropy. Since the scalar field has a Compton frequency $f \approx 2×10^{-8}$ Hz for $m\_a=10^{-22}$ eV​file-pbs5tcrmsvz7ndprsed51h, we anticipate oscillatory signals on timescales of years. The emitted waves in the simulation manifest as expanding spherical shells of lighter-density fluctuation that carry away excess mass and energy, leaving behind a smaller, stable soliton. This is analogous to an “axion star bosenova.” We will measure what fraction of the core’s mass is radiated and how far the waves travel (likely dispersing into the halo). These results allow us to predict possible **transient signals**: for example, a sudden density spike and wave burst in the dark matter could induce a small change in the gravitational potential.

Importantly, any rapid mass redistribution can source **gravitational waves (GWs)**. Although the bulk of energy goes into scalar field oscillations, a fraction can convert to gravitational radiation. We will use the quadrupole formula to estimate GWs from our collapse simulations. The frequency of such waves would be around the core’s dynamical frequency (which for a dense $\sim10^8 M\_\odot$ core is high, possibly millihertz to hertz). The amplitude is expected to be tiny, but many such events across the Universe (especially at early times when halos form and merge) could superpose into a background. Recent work suggests that the **formation of solitonic “oscillons” in the early universe is accompanied by a stochastic GW background** at frequencies below the soliton formation scale​[arxiv.org](https://arxiv.org/html/2310.03594v2#:~:text=Formation%20of%20cosmological%20solitons%20is,not%20strongly%20interact%20or%20cluster). We will compute whether the cumulative effect of numerous core collapses/mergers could produce a detectable GW background. If the scalaron model is correct, one prediction is a **nanohertz-frequency GW background** from the epoch of first galaxy formation, arising from global oscillations of the field. We will compare the amplitude of this background to current pulsar timing array limits (and detections). This bridges to the next track, where we match these simulation-derived signals to observational data from PTAs and LISA.

Finally, the collapse simulations will inform the **stability criteria** for the scalaron: we will delineate the range of core masses and halo conditions under which solitons remain stable, oscillatory, or unstable. This helps refine RFT 9.0 by indicating whether any additional physics (self-interactions, etc.) is needed to prevent unwanted collapses or, conversely, if such collapses could be an observable feature (e.g. periodic bursts of relativistic axions or gravitational waves that could be sought in cosmological data).

**Observational Dataset Matching**

Having generated concrete predictions from simulations, we now confront them with existing astrophysical data. The goal is to **extract and fit scalaron model predictions against observations** across multiple scales, thereby validating or constraining RFT 9.0. Our strategy spans high-redshift galaxy counts, Galactic stellar streams, strong lensing, and gravitational wave searches. Each serves as a complementary probe of the scalaron’s effects:

*JWST’s deep-field view of countless distant galaxies (and the nearby WLM dwarf galaxy in the foreground) exemplifies the rich dataset on early galaxy populations now available. The Webb telescope can resolve extremely faint galaxies, allowing us to test whether the scalaron model’s suppression of small halos matches the observed galaxy abundance at high redshifts​*[*esawebb.org*](https://esawebb.org/images/WLMb/#:~:text=This%20image%20shows%20a%20portion,stars%20outside%20the%20Milky%20Way)*​file-pbs5tcrmsvz7ndprsed51h.*

**High-Redshift Galaxy Counts (JWST)**

The **James Webb Space Telescope** has opened a new window onto galaxies in the epoch $z \sim 7$–13, providing a crucial test of small-scale structure. In $\Lambda$CDM, structure formation is bottom-up, producing abundant low-mass halos that merge into larger galaxies. The scalaron (fuzzy dark matter) model, by contrast, predicts a **suppression of low-mass halos** due to the quantum Jeans scale (de Broglie wavelength $\sim$ kpc) smoothing out fluctuations below a certain mass scale​file-pbs5tcrmsvz7ndprsed51h. This should lead to a **turnover or flattening** in the galaxy luminosity function at the faint end (i.e. a deficit of ultra-faint high-$z$ galaxies)​file-pbs5tcrmsvz7ndprsed51h​file-9mbj5mvtgwzrvhmxttwfgp. We use JWST deep-field galaxy surveys to search for this signature. Specifically, we examine the UV luminosity function and stellar mass function of galaxies in the range $7 \lesssim z \lesssim 10$​file-pbs5tcrmsvz7ndprsed51h. Our simulations predict that if $m\_a \sim 10^{-22}$ eV, halos below roughly $M\_{\rm halo}\sim10^8$–$10^9,M\_\odot$ will be scarce​file-pbs5tcrmsvz7ndprsed51h. These halos correspond to very faint galaxies (absolute UV magnitude $M\_{\rm UV} > -15$). JWST can detect galaxies down to $M\_{\rm UV}\approx -16$ or fainter at those redshifts, so it is finally probing the regime where a fuzzy dark matter cutoff might appear​file-pbs5tcrmsvz7ndprsed51h​file-pbs5tcrmsvz7ndprsed51h.

Intriguingly, **early JWST results show a surprisingly high abundance of bright galaxies at $z > 7$** – in fact, higher than some $\Lambda$CDM models had forecast​file-9mbj5mvtgwzrvhmxttwfgp. There are galaxies with stellar masses of order $10^8$–$10^9 M\_\odot$ already in place by $z\sim10$. This appears to conflict with a naive fuzzy DM expectation of *delayed* galaxy formation; however, there is an important subtlety. If fuzzy DM suppresses the number of small halos, it can actually *channel more matter into a fewer larger halos* early on, potentially **enhancing the formation of massive galaxies** (since there are fewer small halos competing for the matter)​file-pbs5tcrmsvz7ndprsed51h​file-pbs5tcrmsvz7ndprsed51h. In other words, one can reconcile JWST’s abundant big galaxies with a scalaron model if star-formation efficiency is high in those halos and the lack of small halos means the Universe’s budget of baryons went into the big ones. Indeed, one analysis found that an axion mass of $m\_a \approx 5×10^{-23}$ eV could reproduce JWST’s observed galaxy density at $z\sim8$ while also satisfying reionization constraints, by **suppressing smaller galaxies** so that the observed ones form stars more efficiently​file-pbs5tcrmsvz7ndprsed51h. This scenario implies a **fuzzy DM cutoff** that reduces excess low-mass sources of UV photons, thus keeping reionization on track with fewer, bigger galaxies​file-9mbj5mvtgwzrvhmxttwfgp​file-pbs5tcrmsvz7ndprsed51h.

We will quantitatively fit our model to the **JWST luminosity functions**. Using the halo mass functions from our simulations (for various $m\_a$), we populate them with a simple galaxy luminosity model and compare to JWST counts. The scalaron mass will be treated as a parameter to be constrained. If the data show **no turnover down to the observational limit**, it implies that structure is not suppressed at least down to the corresponding halo mass. For example, JWST and HST have not yet seen a clear drop-off in galaxy counts at faint magnitudes; this pushes the fuzzy dark matter mass **floor** upward (since a higher $m\_a$ has a smaller suppressed regime). If JWST finds **abundant low-mass galaxies or star formation** at $z>10$ that exceed what any $m \sim 10^{-22}$–$10^{-23}$ eV model can produce, those low masses would be ruled out​file-pbs5tcrmsvz7ndprsed51h. In contrast, if JWST were to detect a **deficit of faint galaxies** or an early cutoff in the luminosity function relative to $\Lambda$CDM expectations, it would strongly favor the scalaron model​file-pbs5tcrmsvz7ndprsed51h. Our approach is to compute the predicted galaxy counts for each $m\_a$ and perform a likelihood analysis against the JWST observations. This yields a **best-fit scalaron mass** (or lower limit). For instance, we might find that $m\_a > 2×10^{-21}$ eV is required to avoid under-predicting the number of $M\_{\rm UV}\approx -17$ galaxies at $z=9$. On the other hand, the presence of extremely massive galaxies at $z\sim10$ might pull the fit toward a lower $m\_a$ (since some suppression of small halos can help produce big early halos without violating reionization limits​file-pbs5tcrmsvz7ndprsed51h​file-pbs5tcrmsvz7ndprsed51h). We will also check JWST’s measurements of **galaxy sizes and internal structure**: fuzzy dark matter predicts that small galaxies may be more diffuse with larger cores and fewer compact clumps of stars​file-pbs5tcrmsvz7ndprsed51h. Any indication that high-$z$ dwarfs have puffier profiles than expected could corroborate the scalaron’s influence.

In summary, the JWST dataset will allow us to either **falsify** certain scalaron mass ranges (if they over- or under-predict high-$z$ galaxies)​file-pbs5tcrmsvz7ndprsed51h or find a niche where the model matches JWST and other constraints. Already, hints of tension between JWST’s massive galaxies and standard CDM provide motivation to explore $m\_a \sim 10^{-22}$–$10^{-23}$ eV models​file-pbs5tcrmsvz7ndprsed51h​file-pbs5tcrmsvz7ndprsed51h. We will document whether a single $m\_a$ can simultaneously account for JWST’s observed abundance of galaxies and their inferred contribution to reionization, thereby sharpening the viability of RFT 9.0.

*Simulations of early galaxy formation under different dark matter models.* ***Top:*** *In Cold Dark Matter, numerous small clumps form (bright spots) that later merge.* ***Middle:*** *Warm Dark Matter produces filamentary structures with fewer small halos.* ***Bottom:*** *Fuzzy Dark Matter (scalaron) also forms filaments but with prominent* ***wave interference ripples*** *(“striations”) along them, like the strings of a harp​*[*news.mit.edu*](https://news.mit.edu/2019/early-galaxy-fuzzy-universe-simulation-1003#:~:text=Caption%3A%20A%20simulation%20of%20early,the%20strings%20of%20a%20harp)*. These interference patterns are a distinctive prediction of the scalaron model that can affect galaxy formation in subtle ways.*

**Stellar Stream Perturbations (Gaia)**

In the **Milky Way halo**, long, cold stellar streams offer an exceptionally sensitive probe of dark matter’s granularity. Thin streams, such as those from the Palomar 5 globular cluster and the GD-1 stream, act like **tape-recorders** of past gravitational encounters. In a $\Lambda$CDM universe, streams are periodically tugged by passing subhalos, creating localized gaps or offsets. In the scalaron scenario, there are far fewer subhalos, but the dark matter halo itself is filled with a **time-varying granular density field** from interference of the scalar wavefunction​file-pbs5tcrmsvz7ndprsed51h. This means even without subhalos, streams might experience persistent, low-level fluctuations. We use **Gaia DR3** data (and upcoming DR4) which provide precise positions and velocities for stars in these streams, to search for the imprints of the scalaron dark matter.

Our simulation of streams (described above) yields concrete expectations: fuzzy dark matter of mass ~$10^{-22}$ eV produces granular clumps of mass $\sim10^6$–$10^7 M\_\odot$ that drift through the halo​file-pbs5tcrmsvz7ndprsed51h. As a stream orbit, it will encounter many such “wave nodes.” The cumulative effect is a **smooth perturbation** of the stream, causing it to thicken slightly and develop low-amplitude oscillations in stellar density​file-pbs5tcrmsvz7ndprsed51h​file-pbs5tcrmsvz7ndprsed51h. This is distinct from the CDM case where one expects occasional **sharp gaps or missing segments** from a rare encounter with a dense subhalo. We will measure the stream’s density and width profile and perform a power spectrum analysis of deviations along its length​file-pbs5tcrmsvz7ndprsed51h​file-pbs5tcrmsvz7ndprsed51h. A key discriminator is the **frequency spectrum of perturbations**​file-pbs5tcrmsvz7ndprsed51h: Fuzzy DM induces a spectrum with power at the de Broglie scale (order of a few kpc) and a relatively continuous distribution of low-frequency modes (because many waves overlap). CDM subhalo hits, in contrast, produce isolated kicks corresponding to a localized impulse, adding power at a specific scale (the gap size) but not a continuous wobble.

From current observations: Palomar 5’s stream is extremely thin and coherent, and GD-1, while showing some spur and gap features, mostly lacks evidence for multiple heavy subhalo interactions. We will compare the measured properties to our model: If the streams are **too pristine** (no significant fluctuations beyond what baryonic effects like disk tides can explain), we can set a **lower bound on $m\_a$**. For example, earlier analyses (pre-Gaia) of several streams already suggested $m\_a > 1.5×10^{-22}$ eV at 95% confidence​file-pbs5tcrmsvz7ndprsed51h, otherwise lighter (more fuzzy) DM would have perturbed the streams more than observed. With Gaia’s improved data, if Pal 5 is still knife-thin over its length, we might push this limit higher – perhaps excluding even $m\_a = 2×10^{-22}$ eV if the stream shows **no sign of granularity**​file-pbs5tcrmsvz7ndprsed51h. We will rigorously derive this by injecting our simulated perturbation profiles into mock streams and determining what amplitude of fluctuations would be detectable. Conversely, if we do find unexplained disturbances in the streams – subtle density wiggles or an unexpectedly high residual velocity dispersion – we will check if their properties match the fuzzy DM prediction (continuous, low-amplitude perturbations)​file-pbs5tcrmsvz7ndprsed51h​file-pbs5tcrmsvz7ndprsed51h. A **positive detection** of the predicted pattern, especially in multiple streams with a consistent amplitude, would be strong evidence **bolstering the scalaron model​file-pbs5tcrmsvz7ndprsed51h**. We will need to rule out other causes (e.g. encounters with known massive perturbers like the Galactic bar or molecular clouds) to claim such a detection.

Thus, stellar streams serve as a **yes/no test** of the scalaron’s grainy halo effect: the absence of the expected stream perturbations would challenge the model (implying $m\_a$ must be so high that the granules are too small to matter), whereas finding the right kind of oscillatory perturbation would provide a novel confirmation​file-pbs5tcrmsvz7ndprsed51h. Our work will produce an updated constraint: e.g. “Based on Pal 5 and GD-1, we find no evidence of fuzzy perturbations, setting $m\_a > 2.0×10^{-22}$ eV (95% CL),” or, in the hopeful scenario, “We detect a subtle wavy distortion in Pal 5 consistent with $m\_a \approx 1×10^{-22}$ eV.” This will directly inform the viability of RFT 9.0 on Galactic scales. Notably, the **tolerance for null results** is built into our falsification criteria – we will define quantitatively what level of stream disruption would rule out the model​file-pbs5tcrmsvz7ndprsed51h.

**Strong Gravitational Lensing Substructures**

Strong lensing offers a **gravitational microscope** to detect dark matter clumps via their effect on lensed images. When a distant quasar or galaxy is strongly lensed by a foreground galaxy, small dark subhalos in the lens can perturb the lensed images (e.g. causing anomalous flux ratios or slight image position shifts). This method is sensitive to subhalos well below the detection limit of direct imaging, down to masses of ~$10^7$–$10^8 M\_\odot$ in favorable cases. It has become a key test of the CDM prediction of numerous subhalos. For the scalaron model, a clear prediction is a **paucity or absence of subhalos below the cutoff scale** – effectively, if $m\_a$ is $10^{-22}$ eV, halos below $\sim10^8 M\_\odot$ are heavily suppressed​file-pbs5tcrmsvz7ndprsed51h. Thus, lensing should find fewer subhalos than CDM would have, or none below a certain mass. We compile data from surveys of strong lens systems (including recent JWST high-resolution imaging of lenses​[arxiv.org](https://arxiv.org/abs/2410.12987#:~:text=arXiv%20arxiv,NIRCam%20imaging%20across%20multiple%20filters) and ALMA observations of lensed arcs at radio/mm wavelengths). These analyses often report constraints on the subhalo mass function or directly give limits on alternative DM models.

Our approach is to take the **subhalo populations from our simulations** (for each trial $m\_a$) and use them to predict lensing signatures. For example, we can simulate lensing of a background galaxy through a foreground halo that contains the subhalo distribution from a fuzzy DM simulation, and compare it to a CDM case. The scalaron model might produce noticeably **smoother lenses** – fewer small perturbations in the Einstein ring or in the brightness of multiple images. Observationally, lensing studies have so far found some substructure. For instance, analyses of multiple quasar lenses and one extended arc have detected the equivalent of a few $10^8$–$10^9 M\_\odot$ subhalos, consistent with CDM expectations at that scale. These detections already imply dark matter cannot erase all small structure above ~10^8 M☉. In particular, a recent study of a lensed arc placed a lower bound **$m\_a > 4.4×10^{-21}$ eV** (ruling out $m\_a \le 4.4×10^{-21}$ at 20:1 odds)​ui.adsabs.harvard.edu. This is quite a stringent constraint, suggesting that if the scalaron is the sole dark matter, it must be heavier than a few $10^{-21}$ eV to not overly deplete the subhalo population. Furthermore, the absence of lensing anomalies on very small scales and the success of CDM-based lens models indicate no dramatic cutoff in the $\sim10^8$–$10^9 M\_\odot$ range – pushing the cutoff (if any) to lower masses.

We will use these observations to **fit the scalaron mass**. Essentially, the number of subhalos (or lens perturbation amplitude) as a function of $m\_a$ will be compared to what lensing datasets demand. If, for example, our model with $m\_a = 10^{-22}$ eV produces on average 0 subhalos of $>10^8 M\_\odot$ in a Milky Way-mass halo (which is too extreme), whereas lensing clearly finds a few such subhalos per halo, then $10^{-22}$ eV is disfavored. We expect that to satisfy lensing, **$m\_a$ likely needs to be $\gtrsim 10^{-21}$ eV**​file-9mbj5mvtgwzrvhmxttwfgp. We will formalize this by computing the likelihood of the lensing data given each $m\_a$. The outcome will be a probabilistic constraint — e.g. $m\_a > 1×10^{-21}$ eV at 95% confidence, consistent with earlier inferences that lensing and Lyman-$\alpha$ forest data push toward heavier axion masses​file-9mbj5mvtgwzrvhmxttwfgp​file-pbs5tcrmsvz7ndprsed51h.

Additionally, we will search for **qualitative differences** in lensing that could distinguish a wave-dominated halo from a particle-dominated one. One possibility is *diffractive lensing*: if light passes through a fuzzy core, the coherence of the scalar field could induce subtle wave-optics effects (like a blur or fringe) rather than the sharp gravitational deflections of a cuspy core​file-9mbj5mvtgwzrvhmxttwfgp. While current data is not quite at the resolution to see this, future extremely high-res imaging (e.g. VLBI radio observations of lenses​[arxiv.org](https://arxiv.org/pdf/2302.10941#:~:text=arc%20arxiv,Key%20words%3A)) might. For now, our focus is on subhalo counts. We will incorporate **line-of-sight structures** as well, since background structure along the sightline can also cause lens perturbations and must be disentangled from true subhalos. By combining results from multiple lens systems, we aim to either find a consistent $m\_a$ that fits all (if the scalaron model is viable) or to conclude that a single $m\_a$ cannot simultaneously satisfy lensing and other observations (thus requiring either an admixture of CDM or ruling out the model).

In summary, strong lensing provides a **crucial check** on RFT 9.0: it ensures that the model isn’t *too “fuzzy”* such that it contradicts the gravitational imprint of small-scale structure that we do observe. Our analysis will likely yield a *lower bound* on the scalaron mass (around the $10^{-21}$ eV scale)​file-pbs5tcrmsvz7ndprsed51h​file-9mbj5mvtgwzrvhmxttwfgp that the model **must respect** to remain viable.

**Pulsar Timing Array and LISA Signals**

One of the most novel aspects of the ultralight scalaron model is the possibility of **direct detection** of its oscillatory dynamics through gravitational waves or precision timing. Unlike WIMP dark matter, which is practically static on human timescales, an axion-like field with $m\_a \sim 10^{-22}$–$10^{-21}$ eV oscillates coherently with a period of order years to months​file-pbs5tcrmsvz7ndprsed51h. This opens two detection channels: **pulsar timing arrays (PTAs)** that are sensitive to nanohertz gravitational waves (periods of years), and space-based laser interferometers like **LISA** that target millihertz frequencies (periods of minutes to hours). We examine both for potential scalaron-induced signals.

First, consider **pulsar timing arrays**. PTA collaborations (NANOGrav, EPTA, PPTA, CPTA) have recently reported evidence for a stochastic common-spectrum process consistent with a gravitational wave background in the nHz band (approximately $f \sim 10^{-8}$ Hz)​[arxiv.org](https://arxiv.org/html/2310.03594v2#:~:text=Recently%2C%20Pulsar%20Timing%20Array%20,While%20the). The conventional interpretation is a background of merging supermassive black hole binaries. However, there are alternative explanations – and one exciting idea is that this could be a signal from new physics in the early universe. **Formation of solitons (oscillons) in the early cosmos can generate a universal gravitational wave background at frequencies below the characteristic scale of non-linear structure formation**​[arxiv.org](https://arxiv.org/html/2310.03594v2#:~:text=Formation%20of%20cosmological%20solitons%20is,not%20strongly%20interact%20or%20cluster). In particular, a recent study demonstrated that axion-like particle soliton formation can produce a PTA-detectable GW spectrum and even noted that the NANOGrav 15-year data show a slight preference for scenarios where these solitons form but do not cluster too strongly​[arxiv.org](https://arxiv.org/html/2310.03594v2#:~:text=Formation%20of%20cosmological%20solitons%20is,not%20strongly%20interact%20or%20cluster)​[arxiv.org](https://arxiv.org/html/2310.03594v2#:~:text=solitonic%20oscillons%20provides%20a%20viable,not%20strongly%20interact%20or%20cluster). We will take our findings from the **collapse simulations** and early structure formation to predict the **amplitude of gravitational waves** generated by the scalaron field. This includes GWs from: (a) the violent birth of solitonic halos (as the scalar field fragments and some overdensities oscillate and radiate), and (b) oscillations or instabilities of solitons (e.g. the collapse events discussed). We will compare the model’s GW spectrum to the PTA data. Key parameters are the GW strain amplitude $A\_{\rm gw}$ and spectral slope. If the scalaron model (for a given $m\_a$) naturally produces an $A\_{\rm gw} \sim 10^{-15}$ at $f\sim10^{-8}$ Hz (roughly the level NANOGrav reports) with a spectrum matching the observed one, this would be a remarkable success – indicating the dark matter and the new PTA signal share a common origin​[arxiv.org](https://arxiv.org/html/2310.03594v2#:~:text=Formation%20of%20cosmological%20solitons%20is,not%20strongly%20interact%20or%20cluster). If, on the other hand, the model overshoots (too much GW power would have been generated, which PTAs would detect) or undershoots (no significant GW background, meaning the PTA signal must be from something else), we will document that. The nice feature is that **PTA data can set an upper bound on how much “activity” the ultralight field had during structure formation**, thereby constraining scenarios with extremely early or energetic soliton formation. For instance, if no stochastic background had been seen, we could use PTA non-detections to say “the scalaron field’s collapse events must have been mild (no more than X% of dark matter energy converted to GWs).” But with a detection in hand, we explore if a consistent story can be told where that detection is actually a sign of the scalaron. This involves cross-checking with other cosmological probes (e.g. the model must not overproduce primordial black holes either, which the axion scenario nicely avoids​[arxiv.org](https://arxiv.org/html/2310.03594v2#:~:text=demonstrate%20that%20universal%20GW%20background,not%20strongly%20interact%20or%20cluster)).

Now, for **LISA (Laser Interferometer Space Antenna)**, the situation is more speculative. The scalaron field oscillation frequency for our fiducial mass ($10^{-22}$ eV) is far below LISA’s band – on the order of $10^{-8}$–$10^{-7}$ Hz​file-pbs5tcrmsvz7ndprsed51h, whereas LISA is sensitive to $10^{-4}$–$10^{-1}$ Hz. However, if the scalaron mass were at the upper edge of our consideration ($\sim10^{-20}$ eV), the oscillation frequency would be $\sim10^{-6}$ Hz, creeping into the milliHertz range. Moreover, any **transient events** like soliton mergers or collapses could generate a burst of higher-frequency gravitational waves. We will search our collapse simulations for any high-frequency components – for example, the core oscillation during collapse might have a characteristic frequency related to the free-fall time of the core (which for a $10^8 M\_\odot$ soliton is perhaps $10^{-5}$–$10^{-4}$ Hz). If there is a mechanism to up-convert some power to those frequencies (perhaps via nonlinear interactions or a smaller sub-soliton structure), then in principle LISA could detect it. In addition, the scalar field might affect LISA’s arm-length measurements if it has a coupling to fundamental constants (though our focus in RFT 9.0 is gravitational effects). While a direct LISA detection of fuzzy dark matter oscillations is unlikely for $m \sim 10^{-22}$ eV, we include LISA in our considerations as a **future probe of any faster oscillations or exotic events**. For instance, if an ultra-dense axion mini-halo (an “axion star”) of intermediate mass ($\sim10^5 M\_\odot$) were to collide with a black hole or tidally disrupt, the burst of GWs might fall into LISA’s range. We will estimate event rates for such occurrences – though admittedly speculative, it’s part of covering all bases for potential empirical signs of the scalaron model.

Lastly, we note there are **laboratory searches** for oscillating fields as well. While not asked explicitly in this track, it’s worth mentioning that atomic clock networks on Earth and in space are being used to detect tiny oscillations in fundamental constants that an ultralight scalar would induce​file-pbs5tcrmsvz7ndprsed51h. For example, comparisons of atomic clocks (on GPS satellites vs ground, or between distant laboratories) have been used to search for a common-frequency oscillation that could be the dark matter wave passing through​file-pbs5tcrmsvz7ndprsed51h​file-pbs5tcrmsvz7ndprsed51h. These experiments are sensitive to the same frequency range as PTAs (nanohertz to microhertz). A null result in multi-year clock comparisons can set limits on the local amplitude of scalar dark matter oscillations. We will incorporate any such findings to cross-check the astrophysical constraints. If, say, an atomic clock array were to see a hint of a $10^{-8}$ Hz oscillation, it could be correlated with the phase of oscillations expected in our simulated halo (perhaps linking to a field coherence across the solar neighborhood). This interdisciplinary approach strengthens the validation of the model: **astrophysical and laboratory limits together** can pin down the allowed properties of the scalaron.

**Combined Constraints and Model Likelihood**

By synthesizing the results from all the above observational probes, we aim to **refine the parameter space and likelihood of the Adaptive Scalaron Model (RFT 9.0)**. Each dataset constrains the scalaron (axion) mass $m\_a$ and other aspects (like soliton core density) in different ways, so a consistent model must navigate all of them. We will integrate the constraints as follows:

* **JWST High-$z$ Galaxies:** Indicates possible preference for *lower* $m\_a$ (around a few $\times10^{-23}$ eV) to explain suppressed small-halo counts yet high massive galaxy abundance​file-pbs5tcrmsvz7ndprsed51h​file-pbs5tcrmsvz7ndprsed51h – but requires that $m\_a$ not be so low as to conflict with other data​file-pbs5tcrmsvz7ndprsed51h.
* **Stellar Streams:** Demand $m\_a$ above $\sim10^{-22}$ eV (or else streams would be hotter/thicker than observed)​file-pbs5tcrmsvz7ndprsed51h​file-pbs5tcrmsvz7ndprsed51h. Non-detection of stream perturbations pushes $m\_a$ to the higher end, whereas a detected fluctuation could pinpoint $m\_a$ near $10^{-22}$ eV.
* **Strong Lensing & Ly-$\alpha$:** Strongly favor *higher* $m\_a$ (roughly $>10^{-21}$ eV) to retain enough small-scale power​file-pbs5tcrmsvz7ndprsed51h​file-9mbj5mvtgwzrvhmxttwfgp. One lensing analysis gave $m\_a >4.4×10^{-21}$ eV​ui.adsabs.harvard.edu, implying the scalaron must behave almost like CDM on sub-galactic scales.
* **PTA Gravitational Waves:** The NANOGrav signal could be fit by $m\_a$ in a certain range if the timing matches soliton formation; this might prefer a scenario with intermediate $m\_a$ and specific early-universe conditions​[arxiv.org](https://arxiv.org/html/2310.03594v2#:~:text=Formation%20of%20cosmological%20solitons%20is,not%20strongly%20interact%20or%20cluster). If the scalaron is too heavy, it might produce negligible GW background; if too light, it might produce an overshoot of GW energy or conflicting structure formation history. Current PTA data seem consistent with $m\_a \sim10^{-22}$–$10^{-21}$ eV models (though this is an emerging area of study).

We will perform a **joint likelihood analysis** where we vary $m\_a$ (and potentially a few nuisance parameters like star formation efficiency or the fraction of dark matter that is ultralight if we allow a mixed model) to find what value best fits *all* the data. This essentially treats the above bullets as independent constraints to multiply together. We will obtain a *posterior probability distribution* for $m\_a$. It may turn out, for example, that the posterior peaks around $m\_a \approx 2×10^{-21}$ eV as a compromise: this could moderately suppress dwarfs (helping with cores and JWST to some extent) while still being heavy enough to satisfy lensing and Ly-$\alpha$ forest limits. Or perhaps the peak is bimodal, indicating tension – e.g. one mode at $5×10^{-23}$ eV (to fit JWST) and another at $2×10^{-21}$ eV (to fit lensing), with no single value fitting all. In that case, we might conclude that **no single-mass fuzzy DM can satisfy all current observations** at 95% CL, pointing to either new physics or a mixed dark matter scenario​file-9mbj5mvtgwzrvhmxttwfgp. This would be a critical finding: it would mean RFT 9.0 in its pure form is not viable unless modified.

However, if a **consistent mass range emerges** that threads the needle, it would be a major success for the scalaron model. For instance, suppose $m\_a \approx 1×10^{-21}$ eV is acceptable to lensing (just barely), produces only mild stream perturbations (also acceptable), and with slight tweaks to star formation models can match JWST counts (perhaps requiring efficient star formation in early halos). Then RFT 9.0 would stand as a viable alternative to $\Lambda$CDM, with concrete predictions for upcoming data. We would then calculate the **Bayesian evidence** for the scalaron model vs $\Lambda$CDM. This takes into account the improvements in fit for small-scale issues versus the extra complexity of introducing $m\_a$ as a parameter. If the **model likelihood** is substantially higher than $\Lambda$CDM when all these data are considered (or if $\Lambda$CDM struggles to explain certain observations at all, like a confirmed cutoff in the galaxy LF or large cores in every dwarf), that would strongly motivate the community to take the scalaron hypothesis seriously. On the other hand, if $\Lambda$CDM still provides a comparably good fit (especially once baryonic feedback is accounted for) and the scalaron doesn’t dramatically improve it, the data would favor the simpler CDM explanation.

At present, there are **mixed signals**: JWST and local dwarf galaxy cores show tantalizing deviations that align with fuzzy DM expectations (e.g. fewer small halos, cored profiles)​file-9mbj5mvtgwzrvhmxttwfgp, whereas Lyman-$\alpha$ forest and lensing push the particle mass to be so high that those benefits diminish​file-9mbj5mvtgwzrvhmxttwfgp. It’s possible that the **truth is intermediate**, such as a **partial dark matter scenario** where a fraction of DM is ultralight and the rest is cold, or an ultralight scalar that has additional interactions (self-couplings) that change its small-scale behavior. Our validation effort will clarify if **RFT 9.0 can survive in a “pure” form** or if it needs such modifications. For example, if we find $m\_a \sim 10^{-22}$ eV is best for cores but $10^{-21}$ eV for subhalos, we might suggest a two-component model (some fraction at each mass) to satisfy both – effectively an adaptive version of the scalaron hypothesis.

In conclusion, this deep research initiative uses simulations and observations in concert to stress-test the Adaptive Scalaron Model. The **targeted simulations** yield concrete, testable predictions: solitonic core sizes, halo interference patterns, subhalo survival rates, and possible collapse-induced waves. The **observational matching** then confronts each prediction with data from JWST, Gaia, strong lensing surveys, and gravitational wave experiments. By grounding the model in empirical reality, we refine its parameters (like the scalaron mass) and assess its overall plausibility. The outcome will be a much clearer picture of whether this ultralight “fuzzy” scalar field can serve as a realistic dark matter candidate that **matches our universe’s behavior** – from the tiniest galactic cores, through the presence (or absence) of halo substructure, all the way to potential ripples in space-time itself. If the scalaron model passes these tests, RFT 9.0 will emerge as a robust, realistic theory. If it fails, we will have charted exactly where and why, providing guidance for the next iteration of dark matter theory (be it a different mass, a hybrid model, or a return to baryonic solutions for small-scale issues)​file-pbs5tcrmsvz7ndprsed51h. Such is the scientific process of validation that we undertake: by the end, the scalaron hypothesis will be either strongly bolstered – with a concordant set of simulations and observations – or constrained to the point that alternative explanations must be sought, thereby refining our understanding of cosmic structure formation. The **Adaptive Scalaron Model** is thus brought from theoretical concept toward a concrete, testable framework, tightly woven into the fabric of observational cosmology.